Analysis of Wireless Energy Transfer System Based on 3-D Finite Element Method Including Displacement Current

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Abstract — With the development of electronic technology, there has been increasing interests in research of wireless power transfer method. The finite element method (FEM) is a powerful tool for the numerical simulation of the system. Generally, the resonant frequency of the wireless power transfer system can be up to several megahertz. Due to the presence of inductance and capacitance in the system, eddy current and displacement current become two essential parts in the electromagnetic field distribution. The problem to be addressed is between low and high frequencies. Conventional low frequency method cannot include displacement current, while common high frequency method cannot couple circuit equations with field equations. In this paper, the Maxwell's equations with displacement current are used to describe the Witricity system. It has the potential to couple the circuit equations directly. Through numerical electromagnetic field analysis, the current induced in the receiver loop and the efficiency of the transfer power between the sender and receiver loops can be easily obtained.

I. INTRODUCTION

In the past several years, the wireless power transfer method was been widely discussed because of the surge in the use of autonomous electronic devices. Indeed, power can be transferred between two coils at the same resonant frequency according to Nikola Tesla [1] in the early twentieth century.The wireless power transfer system being studied is named "Witricity" as in wireless electricity [2].

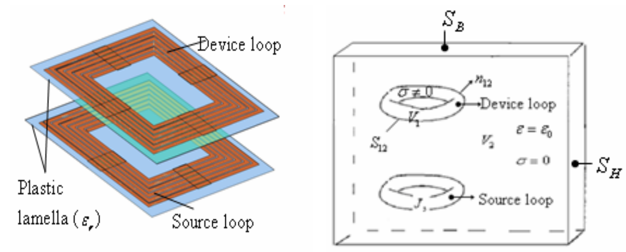
The basic Witricity system consists of two resonators operating with the same resonance frequency; one as the source loop and the other as the device loop. This paper is reporting the study of strongly coupled magnetic resonators by exploring non-radiative magnetic resonant induction at megahertz frequencies. The source loop is coupled inductively to an oscillating circuit and the device loop is coupled to a resistive load.

Numerical solutions of Maxwell's equations have been applied successfully to address many problems in science and engineering. For instance, Maxwell's equation has been used to study problems arising in radar, remote sensing, plasma physics, antennas, high-frequency circuits, etc. In Maxwell's theory, the displacement currents are often deemed to be only of significance at relatively high frequencies, and invariably the case chosen to exemplify their role is in the study of parallel plate capacitor [3].

To study the energy transfer efficiency of the system, the finite element method (FEM) is a very useful tool in numerical simulation. In this paper, the time-dependent Maxwell's equations are solved by the finite element $\vec{A}-\phi$ method [4-5] including displacement current is applied to the analysis of resonating operation of the capacitance and inductance in the Witricity system.

II. NUMERICAL MODEL OF THE SYSTEM

The protocol of the Witricity system is presented as in Fig. 1(a), and its mathematical domain is shown in Fig. 1(b). The domain of the Witricity system V consists of the device loop V_1 which includes conducting objects ($\sigma \neq 0$) but no current source, and the capacitances (ε_r) are also present in this region; the other domain of V is the non-eddy current region V_2 which includes the source loop with current source (J_s). The boundary of V is divided into two parts S_B and S_H where different boundary conditions are imposed: the normal component of the magnetic field intensity on S_B and the tangential component of the magnetic field intensity on S_H are specified separately.



(a) (b)

Fig. 1. (a) The protocol of the Witricity system. (b) The numerical domain of the Witricity system.

The Maxwell's system with displacement current is:

$$\nabla \times \vec{H} = \vec{J}_s + \frac{\partial \vec{D}}{\partial t} \tag{1}$$

$$\nabla \cdot \vec{D} = \rho \tag{2}$$

where; $\vec{H}$ is the magnetic field intensity (A/m); $\vec{J}_s$ is the source current density (A/m^2); $\vec{D}$ is the electric displacement vector; t is the time (s); ρ is the charge density (C/m^3).

The electromagnetic phenomena are described by the following Maxwell's equations:

$$\nu \nabla \times \vec{B} = \vec{J}_s + \varepsilon \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E} \tag{3}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{4}$$

where; $\vec{E}$ is the electric field intensity (V/m); $\vec{B}$ is the magnetic flux density (T); ε, ν, σ are permittivity of the dielectric, the magnetic reluctivity, and electrical conductivity, respectively.

Note that from the equation

$$\nabla \cdot \vec{B} = 0 \tag{5}$$

one can introduce a vector function $\vec{A}$ named the magnetic vector potential such that

$$\nabla \times \vec{A} = \vec{B} \tag{6}$$

Combining (4) and (6), we can obtain

$$\nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0 \tag{7}$$

which implies that there exists a scalar function ϕ named the electric scalar potential such that

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \phi \tag{8}$$

To guarantee the uniqueness of $\vec{A}$, the Coulomb Gauge is applied to the equations. The equations modeling the Wictricity system then become

$$V_1 : \begin{cases} \nabla \times \left(\nu \nabla \times \vec{A} \right) + \sigma \frac{\partial \vec{A}}{\partial t} + \sigma \nabla \phi + \varepsilon \frac{\partial}{\partial t} \left(\frac{\partial \vec{A}}{\partial t} + \nabla \phi \right) = 0 \quad (a) \\ \nabla \cdot \left(-\sigma \frac{\partial \vec{A}}{\partial t} - \sigma \nabla \phi - \varepsilon \frac{\partial}{\partial t} \left(\frac{\partial \vec{A}}{\partial t} + \nabla \phi \right) \right) = 0 \quad (b) \end{cases} \tag{9}$$

$$V_2 : \nabla \times \left(\nu \nabla \times \vec{A} \right) + \sigma \frac{\partial \vec{A}}{\partial t} + \sigma \nabla \phi + \varepsilon \frac{\partial}{\partial t} \left(\frac{\partial \vec{A}}{\partial t} + \nabla \phi \right) = \vec{J}_s \tag{10}$$

$$V : \nabla \cdot \left(-\varepsilon \frac{\partial \vec{A}}{\partial t} - \nabla \phi \right) = \rho \tag{11}$$

$$S_B : \begin{cases} \vec{n} \times \vec{A} = \vec{0} \\ \nu \nabla \cdot \vec{A} = 0 \end{cases} \tag{12}$$

$$S_H : \begin{cases} \vec{n} \cdot \vec{A} = 0 \\ \left(\nu \nabla \times \vec{A} \right) \times \vec{n} = \vec{0} \end{cases} \tag{13}$$

$$S_{12} : \begin{cases} \vec{A}_1 = \vec{A}_2 \\ \nu_1 \nabla \cdot \vec{A}_1 = \nu_2 \nabla \cdot \vec{A}_2 \\ \nu_1 \nabla \times \vec{A}_1 \times \vec{n} = \nu_2 \nabla \times \vec{A}_2 \times \vec{n} \\ \vec{n} \cdot \left(-\sigma \frac{\partial \vec{A}}{\partial t} - \sigma \nabla \phi - \varepsilon \frac{\partial}{\partial t} \left(\frac{\partial \vec{A}}{\partial t} + \nabla \phi \right) \right) = 0 \end{cases} \tag{14}$$

Expanding $\vec{A}$ and ϕ in the basis of the finite element spaces we have:

$$\vec{A} = \sum_{j=1}^{n} A_j \vec{N}_j \tag{15}$$

$$\phi = \sum_{j=1}^{n} \phi_j N_j \tag{16}$$

where n denotes the total number of nodes in the triangulation of V; and N_j, $\vec{N}_j$ are the vector and scalar basis function corresponding to the j-th node, respectively. Let the time variable t to be discretized by the following second-order central differencing schemes with $t_k = k\Delta t$, k=1, 2, …, where Δt is the time step:

$$\frac{\partial \vec{A}}{\partial t} \approx \frac{\vec{A}^{(k+1)} - \vec{A}^{(k-1)}}{2\Delta t} \tag{17}$$

$$\frac{\partial^2 \vec{A}}{\partial t^2} \approx \frac{\vec{A}^{(k+1)} - 2\vec{A}^{(k)} + \vec{A}^{(k-1)}}{(\Delta t)^2} \tag{18}$$

$$\frac{\partial \phi}{\partial t} \approx \frac{\phi^{(k+1)} - \phi^{(k-1)}}{2\Delta t} \tag{19}$$

III. RESULT

From the above formulations, the field of the electromagnetic field can be obtained. Equations (9), (10) and (11) are solved using the FEM for spatial discretization to obtain the distribution of the magnetic field intensity ($\vec{H}$). Fig. 2(a) indicates the mesh result based on the tetrahedron element, and Fig. 2(b) is the result of magnetic field intensity.

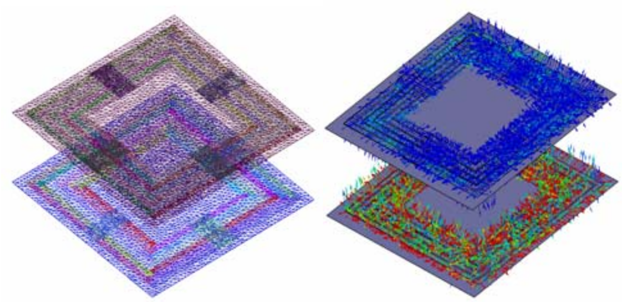


Fig. 2. (a) The mesh of the Witricity system. (b) The magnetic field intentisy of the Witricity system.

The more detailed description of the result will be given in the future full papers.

IV. CONCLUSION

The time-dependent Maxwell's equations by finite element $\vec{A}-\phi$ method including displacement current are applied to simulate the Witricity system. In future research, this method can be used to couple the external circuit directly and it will become more convenient to compute the voltage of the device loop.

V. REFERENCES

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